Effects of including a diffraction term into Rigrod theory for a continuous-wave laser

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Multimode, low-gain continuous-wave lasers are often subject to having intracavity apertures that create diffractive losses inside the optical resonator. For very low-gain systems with short gain lengths, highly reflective mirrors are required to obtain laser oscillation. The Rigrod theory was modified to include a diffractive loss term and comparisons with experimental data show that the intracavity diffractive losses, while small in magnitude, can play a significant role for these low-gain cases with high mirror reflectivities. © 2009 Optical Society of America

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1. Introduction

Rigrod's theories [1–3] of gain saturation and output power have been utilized for over four decades to aid laser scientists in understanding saturation effects in a multitude of different laser systems. Application of Rigrod's original theory [1,2] is straightforward and, through the use of two simple parameters, the unsaturated gain g_o and the saturation parameter w_o, it is possible to obtain an excellent first-order understanding of how laser systems will perform and scale as a function of unsaturated gain and gain length. There are a large number of uses of Rigrod's theories in the literature; applications include the core of unstable cavities [4], lasers with unsaturable losses [3,5–8], and low-gain lasers with large mirror scattering losses [9].

The electrically driven oxygen–iodine laser (ElectricOIL) that was first demonstrated by Carroll *et al.* [10] operates on the electronic transition of the iodine atom at 1315 nm, $I(^2P_{1/2}) \rightarrow I(^2P_{3/2})$ (denoted hereafter as I^* and I respectively). The lasing state I^* is produced by near resonant energy transfer with the singlet oxygen metastable $O_2(a^1\Delta)$ [denoted hereafter as $O_2(a)$]. To date, a 5 cm gain length version of this device has demonstrated a peak small signal gain of $0.0022\,\mathrm{cm}^{-1}$ and a laser power of 28.1 W [11]. One of the significant remaining questions regarding the ElectricOIL system is: why is the amount of laser power extracted so much lower than the available power in the $O_2(a)$ flow? For example, for the case mentioned above, there is 28.1 W of outcoupled laser power for a case that has approximately 350 W of power carried by the $O_2(a)$ flow. Fabry–Perot simulations with the BLAZE-V model [12] indicate that, for this case, we should be able to extract over 130 W in laser power, yet we are only measuring a little over one-fifth of what is predicted. Two possible causes seem the most likely suspects: (i) optical losses that dissipate power before it is outcoupled, and (ii) an unknown chemical kinetic process that is somehow inhibiting (slowing) the power extraction process [13]. In this paper we take a closer examination of the former possibility, i.e., optical losses.

2. Optical Losses

One possibility is that, because we have historically used very high reflectivity mirrors, these optics may have high absorption and/or scattering losses relative to the actual mirror transmission. The mirrors and mounts have not shown any signs of heating during laser operation, so we do not believe that there is

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any significant absorption fraction. Using Rigrod's Eq. 18, Ref. [2] (or, equivalently, related equations from Ref. [9]), it can be shown that it would require an unrealistically large absorption/scattering term to match measured outcoupled power data. The high reflectivity mirrors that we have been using for ElectricOIL experiments (fabricated by AT Films and Los Gatos Research) use superpolished substrates and modern dielectric coating techniques, and we believe that the scattering loss is relatively low (discussed in Subsection 4.A). Thus, we also do not believe that mirror scatter is the predominant effect that is inhibiting the extracted power (as was the case reported by Carroll and Sentman [9] for lower quality mirrors using coating technologies that were 15–20 years old by comparison to today).

However, if we consider Fraunhofer diffraction that occurs off the edges of the supersonic cavity (which effectively act as an aperture), assume uniform filling of the intracavity aperture (which does occur based upon measured beam shapes), then one can make a conservative estimate that the expansion angle is $\sin\theta \cong \theta = \lambda/d = 1.315\,\mu\text{m}/2.54\,\text{cm} = 5.2 \times 10^{-5}$ rad. One can then make an estimate of the amount of beam area from diffractive expansion that could be clipped by the apertures; this estimate comes out to be a loss of approximately 0.00056 per restricting aperture (discussed in more detail in Subsection 4.B). While this number seems, at first glance, to be very small, when one considers the number of passes the photons make through a high reflectivity resonator, then even small intracavity losses may have a significant effect. It is precisely this possibility that we examine in this paper with comparisons to experimental data.

3. Rigrod Derivation Including Diffractive Loss Term

The inclusion of a diffractive loss term δ into the Rigrod equations is fairly straightforward. Since this diffractive loss term occurs outside of the lasing medium, one can essentially incorporate the loss into the mirror reflectance term. However, for completeness, let us begin with Rigrod's original formulation of the problem and include the diffraction term as we solve the equations. Figure 1 illustrates the basic laser setup with two intracavity apertures, and the rise of the normalized intensity, β, as the forward and backward running waves traverse through the laser gain medium.

Following Rigrod's formulation [2], we have

$$\beta_+(z) = \frac{I_+(z)}{I_{\text{sat}}}, \qquad \beta_-(z) = \frac{I_-(z)}{I_{\text{sat}}}, \tag{1}$$

where I is the intensity of the forward or backward running wave and I_{sat} is the saturation intensity parameter (note that we have elected to use I and I_{sat} here, rather than Rigrod's w and w_o notation, but the meaning is the same). The saturated gain can be expressed as

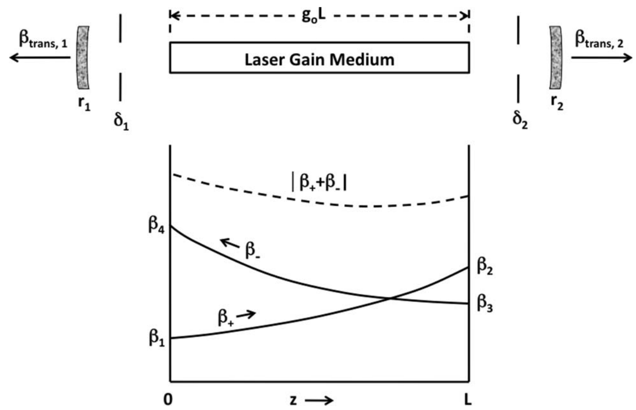

Fig. 1. Schematic diagram of laser with mirrors (r_1 and r_2) and intracavity apertures (δ_1 and δ_2) having unsaturated gain g_o and gain length L. Normalized intensity levels (β) are shown for the forward and backward running waves in an asymmetric laser oscillator.

$$g(z) = \frac{g_o}{(1 + \beta_+ + \beta_-)}, \tag{2}$$

and, since the gain coefficient is assumed to be isotropic, we have

$$g(z) = \frac{1}{\beta_+}\frac{d\beta_+}{dz} = -\frac{1}{\beta_-}\frac{d\beta_-}{dz}, \tag{3}$$

which can be solved to give

$$\beta_+\beta_- = \text{constant} = C. \tag{4}$$

The mirrors have reflectances of r_1 and r_2; however, if we now consider that there is an intracavity aperture loss, then the amount of reflected intensity back into the resonator is reduced by a factor of $1 - \delta$. Therefore, we construct a new "effective" reflectance R that incorporates the diffractive loss term, i.e.,

$$R_1 = r_1(1 - \delta_1), \qquad R_2 = r_2(1 - \delta_2). \tag{5}$$

We can now continue with the derivation. As is obvious from Fig. 1, the reflected intensities are related to the incident ones by $\beta_3 = \beta_2 R_2$ and $\beta_1 = \beta_4 R_1$ or, equivalently,

$$\frac{\beta_2}{\beta_3}R_2 = \frac{\beta_4}{\beta_1}R_1 = 1. \tag{6}$$

From Eq. (4) we get

$$\beta_1\beta_4 = \beta_2\beta_3 = C, \tag{7}$$

and, therefore, Eqs. (6) and (7) can be combined to give

$$\frac{\beta_2}{\beta_4} = \sqrt{\frac{R_1}{R_2}}. \tag{8}$$

Combining Eqs. (2) and (4) leads to

$$g(z) = \frac{g_o}{(1 + \beta_+ + C/\beta_+)}, \tag{9}$$

which can then be integrated to yield

$$g_o L = \ln\left(\frac{\beta_2}{\beta_1}\right) + \beta_2 - \beta_1 - C\left(\frac{1}{\beta_2} - \frac{1}{\beta_1}\right), \tag{10}$$

$$g_o L = \ln\left(\frac{\beta_4}{\beta_3}\right) + \beta_4 - \beta_3 - C\left(\frac{1}{\beta_4} - \frac{1}{\beta_3}\right). \tag{11}$$

Adding Eqs. (10) and (11) and making use of Eqs. (6) and (7) gives

$$2g_o L = \ln\left(\frac{1}{R_1 R_2}\right) + 2[\beta_2(1 - R_2) + \beta_4(1 - R_1)]. \tag{12}$$

Incorporating Eq. (8) and rearranging of terms leads to

$$\beta_2 = \frac{\sqrt{R_1}}{\left(\sqrt{R_1} + \sqrt{R_2}\right)\left(1 - \sqrt{R_1 R_2}\right)} \times \left[g_o L + \ln\sqrt{R_1 R_2}\right], \tag{13}$$

$$\beta_4 = \frac{\sqrt{R_2}}{\left(\sqrt{R_1} + \sqrt{R_2}\right)\left(1 - \sqrt{R_1 R_2}\right)} \times \left[g_o L + \ln\sqrt{R_1 R_2}\right]. \tag{14}$$

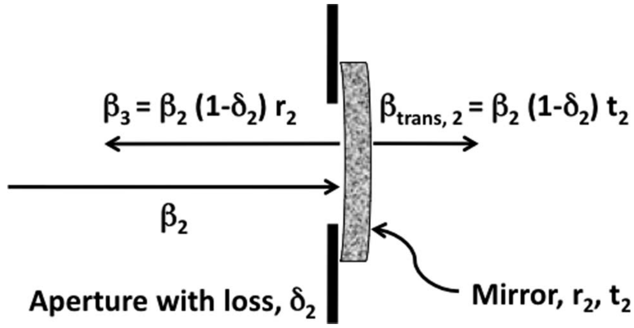

Fig. 2. Schematic diagram showing the effect of an intracavity aperture (δ_2) on the normalized intensity levels (β) transmitted and reflected back into the laser oscillator.

should apply to both the reflected and transmitted waves.

The outcoupled (transmitted) intensity can now be written as

$$I_{\mathrm{out}} = I_{\mathrm{trans},2} + I_{\mathrm{trans},1} = I_{\mathrm{sat}}[\beta_2(1 - \delta_2)t_2 + \beta_4(1 - \delta_1)t_1], \tag{15a}$$

$$= I_{\mathrm{sat}} \frac{(1 - \delta_1)t_1\sqrt{R_2} + (1 - \delta_2)t_2\sqrt{R_1}}{\left(\sqrt{R_1} + \sqrt{R_2}\right)\left(1 - \sqrt{R_1 R_2}\right)} \times \left[g_o L + \ln\sqrt{R_1 R_2}\right]. \tag{15b}$$

Substitution of Eq. (5) into Eq. (15) gives

$$I_{\mathrm{out}} = I_{\mathrm{sat}} \frac{(1 - \delta_1)t_1\sqrt{(1 - \delta_2)r_2} + (1 - \delta_2)t_2\sqrt{(1 - \delta_1)r_1}}{\left(\sqrt{(1 - \delta_1)r_1} + \sqrt{(1 - \delta_2)r_2}\right)\left(1 - \sqrt{(1 - \delta_1)(1 - \delta_2)r_1 r_2}\right)} \left[g_o L + \ln\sqrt{(1 - \delta_1)(1 - \delta_2)r_1 r_2}\right]. \tag{16}$$

We are now ready to consider the outcoupled (transmitted) intensity from the resonator. An interesting question arises here; the diffractive aperture loss term has already been incorporated into the reflectivity term, $R = r(1 - \delta)$, which implicitly lowers the intracavity intensity and, hence, the outcoupled intensity. Is it then appropriate to also include the aperture loss term into the mirror transmission term? To answer this question, consider Fig. 2, where we look at a specialized case of the intracavity aperture being at the edge of the mirror itself (which is, in fact, the case in many laser systems). In this specialized case where the mirror face and aperture essentially share the same plane, it is clear that the δ term

Two specialized cases are of interest. When the diffractive loss is the same on both sides of the gain medium, then $\delta_1 = \delta_2 = \delta$ and Eq. (16) reduces to

$$I_{\mathrm{out}} = I_{\mathrm{sat}} \frac{(1 - \delta)\left[t_1\sqrt{r_2} + t_2\sqrt{r_1}\right]}{\left(\sqrt{r_1} + \sqrt{r_2}\right)\left[1 - (1 - \delta)\sqrt{r_1 r_2}\right]} \times \left[g_o L + \ln\left\{(1 - \delta)\sqrt{r_1 r_2}\right\}\right] \quad \text{for } \delta_1 = \delta_2 = \delta. \tag{17}$$

A second specialized case is that for a symmetrical resonator. For this case, where $r_1 = r_2 = r$, $t_1 = t_2 = t$, and $\delta_1 = \delta_2 = \delta$, then Eq. (16) reduces to

$$I_{\mathrm{out}} = I_{\mathrm{sat}}\left\{\frac{(1-\delta)t}{[1-(1-\delta)r]}\right\}\{g_o L + \ln[(1-\delta)r]\} \quad (18)$$

for a symmetric resonator. As a simple first-order example of the dramatic effect that diffractive loss can have for a high reflectivity mirror case, the term $\{(1-\delta)t/[1-(1-\delta)r]\} = 0.15$ for $r = 0.9999$, $t = 0.0001$, and $\delta = 0.00056$; this indicates that diffraction can potentially account for an 85% (!) reduction in outcoupled intensity (and the situation only gets worse for higher reflectivity mirrors and when mirror absorption/scattering losses are included). This magnitude of power loss could explain most of the difference that we see between expected and measured powers, but perhaps not all of the difference; this will be examined in more detail in Section 4.

The total outcoupled power is simply the outcoupled intensity multiplied by the beam area, A, of the outcoupled beam(s). In other words,

$$P_{\mathrm{out}} = I_{\mathrm{out}} A. \quad (19)$$

Before we explore comparisons of this revised version of Rigrod's theory with experimental data, let us examine the intracavity flux as well as the ratio of diffracted intensity to outcoupled intensity (for reasons that will become apparent in Section 4). The intracavity intensity is, in principle, changing along the optical axis. However, from Eq. (32) of Ref. [2], it can be shown that $\beta \approx$ constant throughout the resonator for high reflectivity mirrors. This is generally a good assumption for any laser system with $g_o L \ll 1$. For the purposes of ElectricOIL, short gain length chemical oxygen–iodine laser (COIL) systems, and hydrogen–fluoride overtone systems we will make this assumption. Thus, the intracavity intensity, β_{IC}, can be taken as approximately equal to any of the β's, and, for the sake of simplicity, we will arbitrarily choose β_2 because it is already defined above in Eq. (13). Substituting in the definition of R_1 and R_2 from Eq. (5) gives

$$\beta_{\mathrm{IC}} \cong \beta_2 = \frac{\sqrt{(1-\delta_1)r_1}}{\left(\sqrt{(1-\delta_1)r_1} + \sqrt{(1-\delta_2)r_2}\right)\left(1 - \sqrt{(1-\delta_1)(1-\delta_2)r_1 r_2}\right)}\left[g_o L + \ln\sqrt{(1-\delta_1)(1-\delta_2)r_1 r_2}\right] \quad (20)$$

for the one-way circulating normalized intensity, and one-way intracavity power is then

$$P_{\mathrm{IC}} = \beta_{\mathrm{IC}} I_{\mathrm{sat}} A. \quad (21)$$

It is interesting to consider the amount of intensity (or power) diffracted to the intensity (or power) outcoupled. We define a parameter ξ to be the ratio of these two values:

$$\xi = \frac{\text{Diffracted Power}}{\text{Outcoupled Power}} = \frac{\beta_2\delta_2 + \beta_4\delta_1}{\beta_2(1-\delta_2)t_2 + \beta_4(1-\delta_1)t_1}. \quad (22)$$

With some algebra, and making use of Eqs. (5) and (8), this expression becomes

$$\xi = \frac{\delta_2\sqrt{(1-\delta_1)r_1} + \delta_1\sqrt{(1-\delta_2)r_2}}{(1-\delta_2)t_2\sqrt{(1-\delta_1)r_1} + (1-\delta_1)t_1\sqrt{(1-\delta_2)r_2}}, \quad (23)$$

which is notably independent of the parameters I_{sat}, g_o, L, and A. For the specialized cases of equal diffractive losses ($\delta_1 = \delta_2 = \delta$) and a symmetric resonator ($r_1 = r_2 = r$, $t_1 = t_2 = t$, and $\delta_1 = \delta_2 = \delta$), respectively, Eq. (23) reduces to

$$\xi_\delta = \frac{\delta\left(\sqrt{r_1} + \sqrt{r_2}\right)}{(1-\delta)\left[t_2\sqrt{r_1} + t_1\sqrt{r_2}\right]}, \quad (24)$$

and

$$\xi_{\delta,r,t} = \frac{\delta}{(1-\delta)}\frac{1}{t}. \quad (25)$$

And, for the case where the absorption/scattering mirror losses are zero, then $t = 1 - r$ and Eq. (25) becomes

$$\xi_{\delta,r,t,a=0} = \frac{\delta}{(1-\delta)}\left(\frac{1}{1-r}\right). \quad (26)$$

For a fixed δ, in the limit as $r \to 1$, it is easy to see that $\xi_{\delta,r,t,a=0} \to \infty$; in other words, the effects of diffractive loss become progressively more severe as the reflectivity approaches unity.

4. Comparisons with Experimental Data

We can now take a serious look at how the addition of a diffractive loss term into Rigrod's theory compares with experimental data. Two gas laser systems will be considered: (i) the VertiCOIL device, which is a COIL system with an extensive data base [14,15], and (ii) the more modern, but still evolving, ElectricOIL system [10,11].

A. Comparisons with VertiCOIL Data

Extensive experimental data were taken for the VertiCOIL device. Hager *et al.* [16] provide an analytic model for estimating gain saturation and power extraction for COIL systems. While the Hager *et al.*

model is quite useful, it is somewhat sensitive to the values of $O_2(a)$ yield and the flow temperature in the laser cavity, which makes it difficult to make estimates of I_{sat} to better than around $\pm 50\%$. Using VertiCOIL information provided by Rittenhouse *et al.* [14], it is possible to use the Hager *et al.* model to estimate the parameter I_{sat} to be in the range of 4100–8200 W/cm^2 (for brevity, the details are not given here). The choice of $I_{sat} = 5100\,W/cm^2$ provides approximately the best fit to the measured power versus reflectivity data when parameters are used from Rittenhouse *et al.*; see Fig. 3. (Note that the data point at $r_1 = 0.842$ was measured to be 0.0, and that the value of gain and gain length suggests laser cutoff at a value of $r_1 = 0.866$). From Fig. 3 it can be observed that there is a flattening of the peak of the curve, as well as a subtle shift of the peak toward lower reflectivity when comparing Rigrod's theory to that including diffraction. Both trends are more consistent with the experimental measurements.

Rittenhouse *et al.* also provide extremely valuable measurements of diffractive spill losses inside the resonator [14]. These measurements were presented as the ratio of diffractive spill to outcoupled power. Using Eq. (23), and the Rittenhouse *et al.* estimation of $\delta_2 = 0.0009$, we show good agreement between the modified Rigrod theory and the measured data; see Fig. 4. The data in Fig. 4 show essentially no dependence on the actual internal aperture used; the modified Rigrod theory including a diffraction term supports this experimental finding.

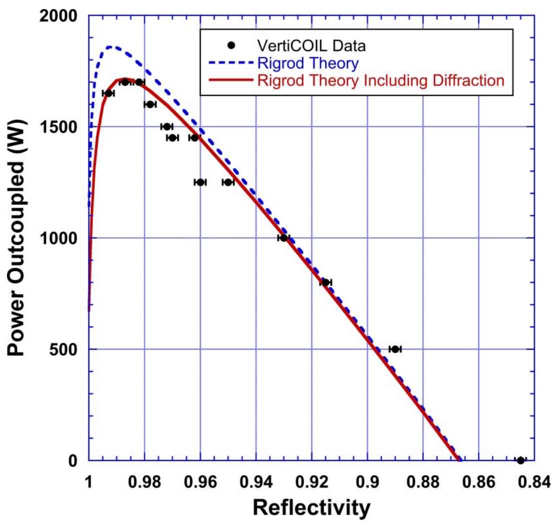

Fig. 3. (Color online) Comparison of original and modified Rigrod theory including a single-pass diffractive loss with experimental VertiCOIL data [14]. Total outcoupled power [using Eqs. (16) and (19)] is plotted as a function of the reflectivity of mirror 1, r_1. Rigrod parameters used were $I_{sat} = 5100\,W/cm^2$, $g_o = 0.0145\,cm^{-1}$, $L = 5.0\,cm$, $A = 5.76\,cm^2$ $(= 1.8\,cm \times 3.2\,cm)$, $r_2 = 0.9987$, $a_1 = a_2 = a = 0.00031$, $t = 1 - r - a$, $\delta_1 = 0.0$, and $\delta_2 = 0.0009$ (with all parameters taken from Rittenhouse *et al.* [14], except for I_{sat}, which was not previously estimated).

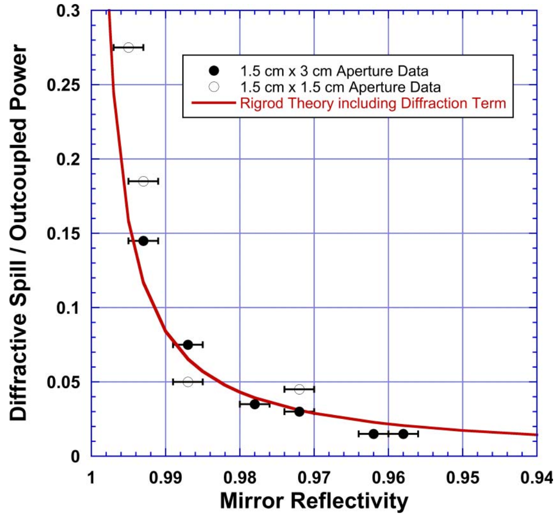

Fig. 4. (Color online) Comparison of modified Rigrod theory including a single-pass diffractive loss with experimental VertiCOIL data [14]. Intracavity diffractive spill to outcoupled laser power [using Eq. (23)], ξ, is plotted as a function of the reflectivity of mirror 1, r_1. Mirror and loss parameters used were $r_2 = 0.9987$, $a_1 = a_2 = a = 0.00031$, $t = 1 - r - a$, $\delta_1 = 0.0$, and $\delta_2 = 0.0009$.

B. Comparisons with ElectricOIL Data

We can now examine the more extreme situation of the ElectricOIL system. In its most recent form [11], ElectricOIL utilizes mirrors each with reflectivities of the order of 0.99–0.9999 and a radius of curvature of 2 m. As mentioned in Section 2, we do not believe that the mirrors used for ElectricOIL experiments have any significant absorption/scattering loss. The first reason for our belief that the ElectriCOIL mirrors have a small absorption/scattering loss is that the manufacturers' specification for this loss was ≤ 0.00003, which was supported by cavity-ring-down spectroscopic measurements of r (for a few selected mirrors) along with measurements of t. Second, let us consider the mirrors used on VertiCOIL that had an absorption/scattering loss of $a = 0.00031$ (Subsection 4.A). Many of the ElectricOIL mirrors have measured transmissivities of $t = 0.00003$, therefore, if they also had a comparable absorption/scattering term to that of the greater than a decade old technology used to coat the VertiCOIL mirrors, then the ratio of a-to-t would be 10-to-1; this would represent a staggering amount of absorption/scattering to transmission. And a third, most important reason, is that the initial ElectricOIL demonstration [10] would not have been possible if the absorption/scattering term was as large as the case for the VertiCOIL mirrors, i.e., with a measured $t = 0.0001$ and an assumed $a = 0.00031$; then $r = 0.99959$, which would require a gain of $0.000082\,cm^{-1}$ to lase, yet the initial lasing experiment was performed with a lower gain of $0.00005\,cm^{-1}$. ElectricOIL lasing was also demonstrated with gains as low as $0.00002\,cm^{-1}$ [17], which necessitates an $r_1 r_2 \geq 0.99980$ (or equivalently

$r_1 = r_2 \geq 0.99990$); for the mirrors used in the experiments of Ref. [17] (having the same coating technology and techniques used for mirrors discussed in this work) the transmission was measured to be 0.000070 (± 0.000001), and we can therefore determine that the absolute upper bound on the absorption/scattering loss is $a \leq 0.00003$ ($= 1.0 - 0.99990 - 0.00007$). Therefore, we strongly believe that the absorption/scattering loss is small and we will assume a value of $a = 0.00002$ for all of the subsequent ElectricOIL calculations.

Let us now try to make an estimate of the value of δ for the ElectricOIL system. At the optical axis location, the height of the nozzle is 2.54 cm. This is the narrowest restriction (aperture) seen by the optical mode. In the horizontal flow direction, there is an aperture that is 4.445 cm in length. From these aperture sizes, one can make a conservative estimate that the expansion angle due to Fraunhofer diffraction in the vertical direction is $\sin\theta_v \approxeq \theta_v = \lambda/d_v = 1.315\,\mu\text{m}/2.54\,\text{cm} = 5.18 \times 10^{-5}\,\text{rad}$, and in the horizontal (flow) direction is $\sin\theta_h \approxeq \theta_h = \lambda/d_h = 1.315\,\mu\text{m}/4.445\,\text{cm} = 2.96 \times 10^{-5}\,\text{rad}$. The length, ℓ, between these apertures is 10.36 cm in the optical axis direction (this distance includes the 5.08 cm gain length laser nozzle plus multiple side plates to hold all of the hardware together); these apertures will confine the beam to be no greater than $2.54\,\text{cm} \times 4.445\,\text{cm}$ in this region. Computing the expansion in one of the vertical directions, $\Delta_v = \theta_v \times \ell = (5.18 \times 10^{-5}\,\text{rad})(10.36\,\text{cm}) = 5.36 \times 10^{-4}\,\text{cm}$, or a total vertical expansion of $2\Delta_v = 1.072 \times 10^{-3}\,\text{cm}$. Along similar lines, computing the expansion in one of the horizontal (flow) directions, $\Delta_h = \theta_h \times \ell = (2.96 \times 10^{-5}\,\text{rad})(10.36\,\text{cm}) = 3.06 \times 10^{-4}\,\text{cm}$, or a total vertical expansion of $2\Delta_h = 0.612 \times 10^{-3}\,\text{cm}$. Now we can make a crude approximation of the clipped multimode beam area to be $A = d_v \times d_h = 11.2903\,\text{cm}^2$, and the unclipped multimode beam area to be $A' = (d_v + 2\Delta_v) \times (d_h + 2\Delta_h) = 11.2966\,\text{cm}^2$; see Fig. 5. Since we have symmetric clipping for both the forward and backward running wave, we can now estimate that $\delta_1 = \delta_2 = \delta = 1 - (A/A') = 1 - (11.2903/11.2966) = 0.00056$ for the ElectricOIL experiments. Note that $\delta_1 + \delta_2 = 0.00112$ for ElectricOIL, which is in reasonable agreement to the Rittenhouse *et al.* [14] estimate of $\delta_2 = 0.0009$ for the total VertiCOIL diffractive loss for a geometry that was reasonably similar.

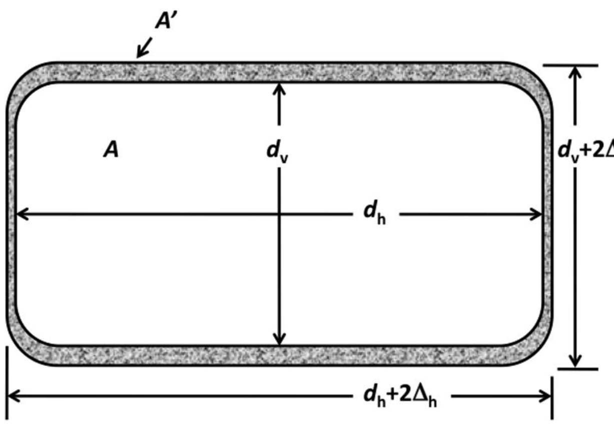

Fig. 5. Illustration of the effect of the aperture that clips off the shaded area, which results from diffraction. Note that the size of the unclipped beam A' is exaggerated.

The Hager *et al.* model [16] can also be applied to ElectricOIL to make estimates of I_{sat}, but as discussed above, it is somewhat sensitive to assumptions about the values of $O_2(a)$ yield and the flow temperature in the laser cavity, thereby making it difficult to obtain estimates of I_{sat} to better than around $\pm 50\%$. Using ElectricOIL information provided by Zimmerman *et al.* [11], along with different assumptions about the $O_2(a)$ yield and iodine flow rates present in the extraction region, it is possible to use the Hager *et al.* model to estimate the parameter I_{sat} to be in the range of 600–1800 W/cm^2 (for brevity, the details are not given here). The beam shape reported in Ref. [11] was not rectangular and we have approximated its area as an ellipse with height and width of 2.54 and 4.44 cm, respectively, having an area of 8.87 cm^2. For the best performing ElectricOIL conditions, the peak gain was measured to be 0.0022 cm^{-1} (repeatability of ± 0.00004 cm^{-1}) at the farthest upstream location in the measurement region, but the average gain as one traverses farther downstream in the flow field is approximately 0.0020 cm^{-1} (the average gain number is thus used for the Rigrod calculations). The choice of $I_{\text{sat}} = 700\,\text{W/cm}^2$ provides approximately the best fit to the measured power versus reflectivity data when other experimentally measured parameters (g_0, L,

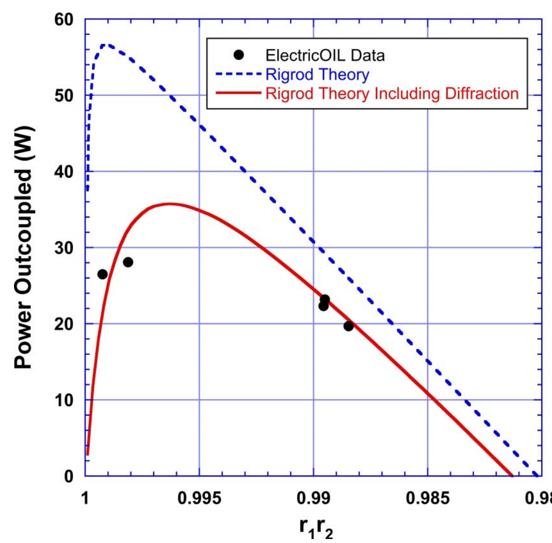

Fig. 6. (Color online) Comparison of original and modified Rigrod theory including diffractive losses with experimental ElectricOIL data [11]. Total outcoupled power [using Eqs. (16) and (19)] is plotted as a function of the product of reflectivities of the two mirrors, $r_1 r_2$. Rigrod parameters used were $I_{\text{sat}} = 700\,\text{W/cm}^2$, $g_o = 0.0020\,\text{cm}^{-1}$, $L = 5.0\,\text{cm}$, $A = 8.87\,\text{cm}^2$, $a_1 = a_2 = a = 0.00002$, $t = 1 - r - a$, and $\delta_1 = \delta_2 = \delta = 0.00056$.

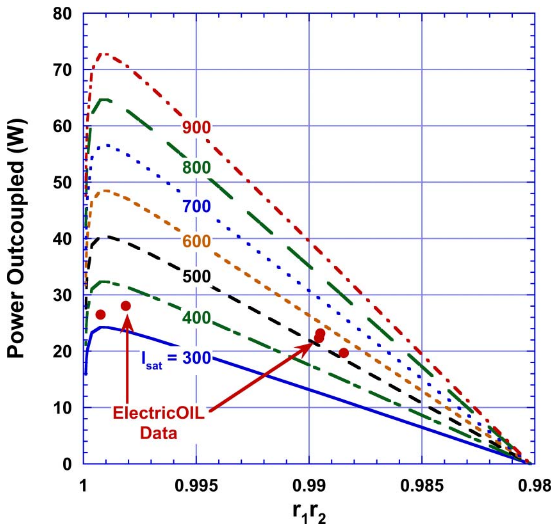

Fig. 7. (Color online) Comparison of experimental ElectricOIL data [11] with original Rigrod theory as I_{sat} is varied. Rigrod parameters used were $g_o = 0.0020\,\text{cm}^{-1}$, $L = 5.0\,\text{cm}$, $A = 8.87\,\text{cm}^2$, $a_1 = a_2 = a = 0.00002$, and $t = 1 - r - a$. Note the diffractive term $\delta_1 = \delta_2 = \delta$ was zeroed.

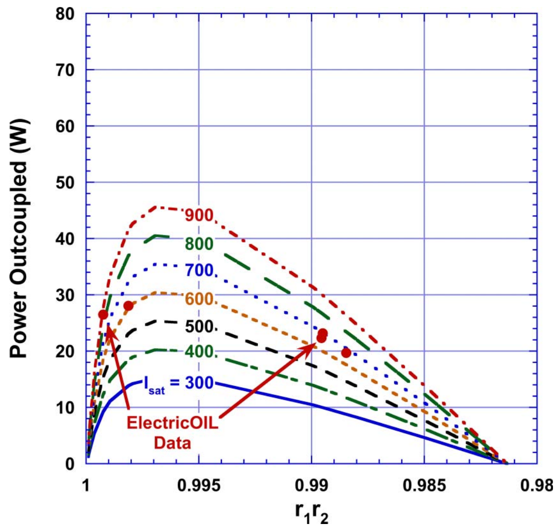

Fig. 9. (Color online) Comparison of experimental ElectricOIL data [11] with modified Rigrod theory including diffractive losses as I_{sat} is varied. Rigrod parameters used were $g_o = 0.0020\,\text{cm}^{-1}$, $L = 5.0\,\text{cm}$, $A = 8.87\,\text{cm}^2$, $a_1 = a_2 = a = 0.00002$, $t = 1 - r - a$, and $\delta_1 = \delta_2 = \delta = 0.00056$.

A, r, and t) are used from Zimmerman *et al.* [11]; see Fig. 6. It is clear that the shape of the original Rigrod theory curve does not adequately represent the experimental data, and that the modified Rigrod theory with diffraction included is in reasonable agreement with the data (a brief parametric variation is provided below). It can be seen that the flattening of the peak from the original theory, as well as the shift in the peak to lower reflectivity, is more pronounced for the ElectricOIL device than for the higher gain VertiCOIL device; see Figs. 3 and 6. Also note that, for the higher gain VertiCOIL case in Fig. 3, that the addition of diffraction resulted in a potential decrease in power of approximately 10% (1870 W peak without diffraction versus 1700 W peak with diffraction); whereas, for ElectricOIL, the power with diffraction is only 61% that of the power without diffraction (35 W peak with diffraction versus 57 W peak without diffraction).

For illustrative purposes, in Figs. 7–10, we provide a sample of how some of the different parameters influence the match between classic Rigrod theory,

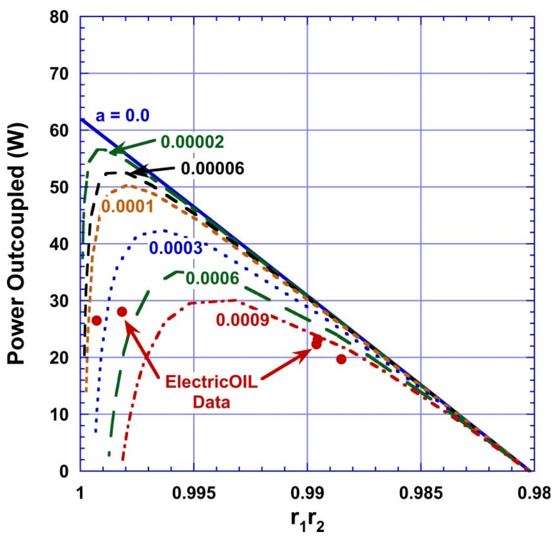

Fig. 8. (Color online) Comparison of experimental ElectricOIL data [11] with original Rigrod theory as the absorption scattering term a is varied. Rigrod parameters used were $I_{\text{sat}} = 700\,\text{W/cm}^2$, $g_o = 0.0020\,\text{cm}^{-1}$, $L = 5.0\,\text{cm}$, $A = 8.87\,\text{cm}^2$, and $t = 1 - r - a$. Note the diffractive term $\delta_1 = \delta_2 = \delta$ was zeroed.

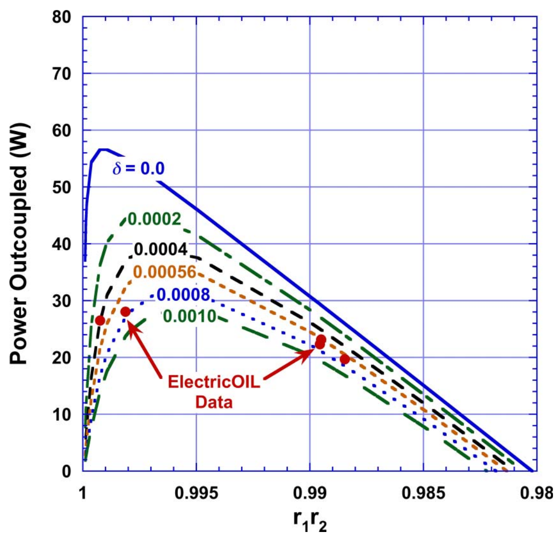

Fig. 10. (Color online) Comparison of experimental ElectricOIL data [11] with original and modified Rigrod theory including diffractive losses as δ is varied. Rigrod parameters used were $I_{\text{sat}} = 700\,\text{W/cm}^2$, $g_o = 0.0020\,\text{cm}^{-1}$, $L = 5.0\,\text{cm}$, $A = 8.87\,\text{cm}^2$, $a_1 = a_2 = a = 0.00002$, and $t = 1 - r - a$.

this modified version of Rigrod theory, and experimental ElectricOIL data. Figure 7 shows that classic Rigrod theory with a variation on I_{sat} and a constant small value of the absorption/scattering term $a = 0.00002$ does not adequately match the experimental data. Similarly, Fig. 8 shows that choosing a constant value of $I_{sat} = 700\,\mathrm{W\,cm^{-2}}$ and varying the absorption/scattering term a using classic Rigrod theory also does not adequately match the experimental data. Note that the variation in absorption/scattering shown in Fig. 8 is far beyond the realm of possibility because of reasons discussed at the beginning of this section, i.e., an upper bound of $a \leq 0.00003$ was established to be consistent with lasing data obtained with very low values of gain of $0.00002\,\mathrm{cm^{-1}}$ [17]. Figure 9 shows that, by using the modified Rigrod theory with a variation on I_{sat} and a constant small value of the absorption/scattering term $a = 0.00002$, and the *a priori* estimated value of $\delta = 0.00056$ (from above), we obtain reasonable agreement with the data with the aforementioned value of $I_{sat} = 700\,\mathrm{W\,cm^{-2}}$. Last, fixing $I_{sat} = 700\,\mathrm{W\,cm^{-2}}$ and the absorption/scattering term $a = 0.00002$, then adjusting the value of the diffractive term δ (Fig. 10) shows that there is less sensitivity to the value of δ than the other parameters. Certainly it would be possible to obtain some reasonable fits with other combinations of I_{sat} and δ, but that gets away from the point of this work, i.e., that diffractive effects can play an important role in short-gain-length—low-gain laser systems.

5. Concluding Remarks

This analysis suggests that diffraction may be playing a significant role in the smaller than expected extracted laser power in the ElectricOIL system. As our gain performance progressively improves, we should be able to perform experiments with lower reflectivity mirrors (presently on order). As the reflectivity of the mirrors decreases, the relative impact of a constant value of δ (determined primarily by the geometry of the resonator) upon outcoupled power is diminished. We note also that a gas flow boundary layer having an absorptive region, rather than a gain region, may act equivalently like a physical aperture. While this work shows the potential significant impact of diffraction upon our power extraction experiments, there are small uncertainties in the accuracy of the mirror r, t, and a values, and the largest unknowns are the values of I_{sat} and δ. Therefore, testing with lower reflectivity mirrors should help to resolve these questions about the effects of diffractive losses in our experiments. Regardless, these results better illustrate the important role that diffraction can play inside the resonators of low-gain systems. As the gain and gain length of such systems are increased, the diffractive effects will become much less pronounced. In other words, as low-gain systems are scaled to larger sizes, they can become dramatically more efficient.

Appendix A: Nomenclature

a, a_i	Absorption/scattering loss of mirror i
A	Beam area or mode size
A'	Beam area or mode size of unclipped diffracted beam
$d_{h,v}$	Size of intracavity aperture in horizontal/flow (h) and vertical (v) directions (cm)
g	Gain coefficient ($\mathrm{cm^{-1}}$)
g_0	Rigrod gain coefficient parameter ($\mathrm{cm^{-1}}$)
I_{out}	Outcoupled intensity ($\mathrm{W\,cm^{-2}}$)
I_{sat}	Rigrod saturation intensity parameter ($\mathrm{W\,cm^{-2}}$)
$I_{+/-}$	Intracavity intensity of forward (+) and backward (−) running waves ($\mathrm{W\,cm^{-2}}$)
ℓ	Length (distance) between intracavity apertures
L	Length of gain medium in the optical axis direction (cm)
P_{IC}	Intracavity power (oneway) (W)
P_{out}	Outcoupled (transmitted) power (W)
r, r_i	Reflectivity of mirror i
R, R_i	Effective reflectivity of mirror i incorporating diffractive loss term δ
t, t_i	Transmissivity of mirror i
$\beta_{+/-}$	Normalized intracavity intensity of forward (+) and backward (−) running waves
β_i	Normalized intracavity intensity of wave i
β_{IC}	Normalized intracavity intensity
δ, δ_i	Diffractive loss term of resonator side i
$\Delta_{h,v}$	Diffractive expansion in horizontal/flow (h) and vertical (v) directions (cm)
$\theta_{h,v}$	Fraunhofer diffractive expansion angle in horizontal/flow (h) and vertical (v) directions
ξ	Ratio of diffracted to outcoupled powers

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