

Effects of a nonhomogeneous gain saturation law on predicted performance of a high-gain and a low-gain laser systems

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A modified Rigrod theory that uses a nonhomogeneous gain saturation law is used to model outcoupled, total, and intracavity power from a high-gain and a low-gain cw HF chemical laser with good accuracy. The homogeneous gain saturation law is found to have significant limitations when used to model total power data over a wide range of threshold gain values. The use of a nonhomogeneous gain saturation law indicates that a gain saturation law parameter of $m = 1.2$ models University of Illinois at Urbana-Champaign supersonic cw HF fundamental laser data more accurately than a homogeneous gain saturation law ($m = 1$). A completely inhomogeneous saturation law ($m = 2$) models University of Illinois at Urbana-Champaign supersonic HF overtone laser data more accurately than a completely homogeneous gain saturation law.

Introduction

One of the basic issues associated with the development of the cw HF $2 \rightarrow 0$ overtone chemical laser is the fraction of the fundamental power that is obtainable in the overtone, i.e., overtone efficiency ϕ , which is defined as

$$\phi = \frac{P_{20}}{(P_F)_{\max}}, \quad (1)$$

where P_{20} is the total overtone power and $(P_F)_{\max}$ is the maximum total fundamental power for the same flow conditions as the overtone data. A simplistic way of estimating the maximum obtainable overtone efficiency is to assume that all of the HF molecules pumped into the second and third vibrational levels, $v = 2$ and $v = 3$, will lase at the overtone wavelengths. The fraction of molecules f_v pumped into the $v = 1$, $v = 2$, and $v = 3$ states are 0.17, 0.55, and 0.28, respectively.¹ Thus the highest overtone efficiency that could possibly be expected is $0.55 + 0.28 = 0.83$, or 83%. A more realistic approximation is to account for the fact that the HF(3) level is rapidly torn apart by collisional decomposition.²⁻⁴ If all of the

HF(3) is assumed to decompose instantly and then is recreated by the pumping reaction as a mixture of HF(1), HF(2), and HF(3), and the decomposition of HF(3) is repeated again and again, then the amount of HF(2) created will be represented by the geometric series $\sum f_2(f_3)^n$, where n goes from 0 to ∞ . For $f_2 = 0.55$ and $f_3 = 0.28$, this summation is approximately 0.764. Thus the highest overtone efficiency that could realistically be expected is approximately 76%. A lower bound on the maximum overtone efficiency can be estimated by assuming that all, but only, the HF molecules initially pumped into the $v = 2$ state will lase. Thus the lower-bound estimate would be 55%. From a practical standpoint, overtone efficiencies in the range 55–76% should be obtainable. Symbols used in this paper are defined in Appendix A.

Experiments performed at TRW⁵⁻⁸ and at Helios, Inc.⁶⁻¹⁰ yielded maximum efficiencies of approximately 60%. More recent overtone experiments performed at the University of Illinois at Urbana-Champaign (UIUC)¹¹⁻¹³ demonstrated efficiencies as high as 70–90%. The range in the UIUC efficiencies is a consequence of supersonic laser (SSL) fundamental data that indicated a maximum fundamental power of 63–76 W [the range in maximum power is from uncertainties in the mirror losses, $\leq 10\%$, and intracavity absorption losses that may have eliminated lasing on the low J lines $P_1(4-6)$ and $P_2(4,5)$, $\leq 10\%$; $63 \text{ W} + 20\% = 76 \text{ W}$; see Refs. 12 and 13]; however, these data were taken with outcouplers having a reflectivity of no greater than 96%. Be-

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cause overtone data were taken with mirrors having reflectivities of 99.7%, a natural question that arises is: How much fundamental power would be obtained with mirrors having as high a reflectivity as possible?

To answer this question we must resort to recently available scaled TRW Alpha Verification Module (AVM) data,¹⁴ for which there were closed-cavity water-cooled mirror data with two 98.4% reflective mirrors (and other lower reflectivity combinations), as shown in Fig. 1. The scaled fundamental AVM data¹⁴ consist of scaled total power as a function of threshold gain g_t , where the threshold gain is computed by using the 62.8-cm geometric gain length L_g of the laser. The AVM data were scaled to the UIUC data by normalizing the AVM data to the AVM power point for $g_t = 0.0045$ and then multiplying the normalized AVM data by the power predicted for the SSL by the detailed kinetic overtone rotational non-equilibrium chemical laser (ORNECL)^{15,16} model for $g_t = 0.0045$. As a way to compare UIUC SSL data with the scaled AVM data, the UIUC SSL geometric gain length of 30 cm was used to compute the threshold gain. Although the two lasers have different gain lengths, a reasonable comparison between the two lasers utilizes the threshold gain [$g_t = -\ln(r_1 r_2)/2L_g$], which is a function of L_g as the independent variable, i.e., the effects of the different gain lengths are included in the comparison. The uncertainty in the reflectivity of the UIUC fundamental mirrors is $\pm 1\%$, the uncertainty in the reflectivity of the TRW fundamental mirrors is $\pm 0.05\%$, and the uncertainty in the reflectivity of the UIUC overtone mirrors is $\pm 0.07\%$. The more accurate measurements for the TRW mirrors and the UIUC overtone mirrors are a consequence of significantly more sophisticated measurement techniques. All of the UIUC power data (fundamental and overtone) represent an average of at least two data points and have a reproducibility of $\pm 10\%$ of the power value. The individual AVM and UIUC SSL power data have accuracies of $\pm 1\%$ of the power value.

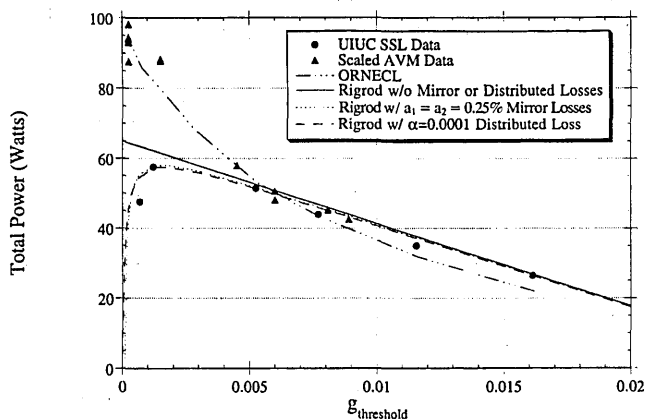


Fig. 1. Fundamental power data compared with predictions by the ORNECL model and homogeneous Rigrod theory as a function of threshold gain. The geometric gain lengths were used to compute g_t .

Figure 1 shows that the water-cooled high-reflectivity (low threshold gain) AVM data curve upward as the threshold gain decreases. The UIUC SSL fundamental data curve downward because the UIUC data points are not corrected for uncertain mirror losses, i.e., the UIUC data represent outcoupled power, not the total power extracted from the laser. Because the AVM data were taken with water-cooled, nontransmissive mirrors, all of the power extracted from the resonator was absorbed by the metal substrate of the mirrors and was calorimetrically measured. Thus the AVM closed-cavity data are total power measurements and do not have any losses for which the data must be corrected. For values of g_t greater than 0.004, the two sets of data are in good agreement with each other. Calculations using the detailed kinetic ORNECL^{15,16} model (which assumed no mirror losses) agreed with the data for all values of threshold gain, i.e., the ORNECL calculations showed an increasing slope as g_t decreased. When the UIUC data for $g_t = 0.0012$ are corrected for mirror and intracavity absorption losses, the estimated maximum power is 76 W, which is in reasonable agreement with the AVM data and the ORNECL prediction. From the scaled AVM data and the ORNECL prediction, the maximum fundamental power of the UIUC SSL may be approximately 94 W (the average of the data points with $g_t = 0.000257$). The highest observed total overtone power (including mirror absorption and scattering losses, i.e., $P_{\text{total}} = P_{\text{transmitted}} + P_{\text{absorbed/scattered}}$) was 64.2 W for the UIUC SSL.^{12,13} If the above estimate for the maximum fundamental power is correct, then the UIUC maximum overtone efficiency becomes 68.3% instead of the 84.5% based on a maximum fundamental power of 76 W. This would bring our overtone efficiencies closer to those measured by TRW.

To answer the question of the maximum overtone efficiency of the UIUC SSL with certainty, one must obtain a better measurement of the maximum fundamental power; this requires the acquisition of high-reflectivity, low-absorption-scattering loss fundamental mirrors that will withstand the thermal stresses of high intracavity powers. The existing experimental setup at UIUC requires a transmissive outcoupler rather than water-cooled optics. Thus the next question that arises is: What is an acceptable set of mirror characteristics (reflectivity/transmissivity/absorption-scattering loss) that will result in 90–100 W of outcoupled fundamental power (power in the bucket)? As a way to address this issue a simple laser oscillator theory, commonly referred to as Rigrod theory,^{17–19} was used to make predictions about laser performance.

Previous research that used homogeneous Rigrod theory produced a straight-line fit to the UIUC SSL data²⁰ when no distributed loss or mirror losses were included as shown in Fig. 1. All of the previous Rigrod theory calculations in Ref. 20 used the effective mixed gain length of 14.9 cm (the calculated gain length of the mixed stream at the center of the mode

rather than the geometric gain length of the laser cavity). As mentioned above, to compare UIUC SSL data with the scaled AVM data, one should use the UIUC SSL geometric gain length of 30 cm instead of the effective mixed gain length of 14.9 cm when computing the threshold gain; because the geometric gain length was used for the AVM data. Thus these calculations were redone by using a 30-cm gain length and by reducing the g_0 parameter from 0.055 cm^{-1} to 0.0275 cm^{-1} . The reason for reducing g_0 by a factor of 2 is that the terms g_0 and L in the Rigrod equations always appear together as the term g_0L ; when L is increased by a factor of 2 ($\sim 30 \text{ cm}/14.9 \text{ cm}$) and g_0 is decreased by a factor of 2, the same value of g_0L is obtained and therefore the same outcoupled powers are obtained for the different mirror reflectivities. The only difference is that the threshold gain values will be a factor of 2 smaller because L has increased by approximately a factor of 2. When a 0.25% mirror absorption-scattering loss was included in the homogeneous Rigrod calculations, Rigrod theory gave a better match to the UIUC SSL data,²⁰ as shown in Fig. 1. However, when the scaled AVM data, which were obtained with water-cooled, high-reflectivity mirrors, are plotted, it is clear that Rigrod theory does not adequately fit the data as g_t decreases (Fig. 1). This leads to the question: Would the inclusion of a nonsaturable uniform distributed loss or a modification to the standard Rigrod theory produce a better fit to the data?

Several papers have investigated the effects of adding a nonsaturable uniform distributed loss^{19,21-24} to the original Rigrod theory. When a distributed loss α_0 of 0.0001 cm^{-1} is used instead of a 0.25% absorption-scattering loss for each mirror, the resulting power versus threshold gain curve is approximately the same as that produced with the 0.25% mirror losses (Fig. 1); there are no significant differences, and either loss mechanism could be used to model the UIUC SSL data. In an attempt to model the upward curvature of the scaled AVM data by using a distributed loss mechanism, a range of values for the parameters g_0 , I_{sat} , and α_0 were tried, but the results gave only a worse agreement with the data and did not produce the upward curvature. The fact that the value $\alpha_0 = 0.0001$ that gave the best agreement with the UIUC SSL data does not equal the value of the mirror losses $\alpha_1 = \alpha_2 = 0.0025$ that gave the best agreement with the UIUC SSL data is not surprising, because the distributed loss and the mirror losses enter the Rigrod formulation in different fashions. Because the mirrors are known to have losses and there is no known distributed loss mechanism for the HF lasers, it is more reasonable to use Rigrod theory with mirror absorption-scattering losses and without distributed loss for the UIUC SSL and the AVM laser. Regardless of which loss mechanism is used, standard Rigrod theory does not adequately fit the scaled AVM data as g_t decreases; the question becomes: Would a modification to the standard Rigrod theory produce a better fit to the data?

To answer this question one must determine which modification is reasonable. One of the *a priori* assumptions with standard Rigrod theory is to use homogeneous gain saturation because the equations can be solved analytically.^{17,18} The justification for using the homogeneous gain law for cw HF chemical lasers is that these devices typically run many longitudinal modes, and Milonni²⁵ showed that the saturation law for multiple modes in an inhomogeneously broadened medium had the same form as the homogeneous gain saturation law in the continuum limit. Table 1 lists the approximate values of the pressure, temperature, Lorentzian (pressure) broadened linewidth, Doppler broadened linewidth, mirror spacing, the spacing of the longitudinal cavity modes, and the likely number of longitudinal modes under the gain curve as a function of laser and wavelength (fundamental or overtone). The fundamental UIUC SSL data can accommodate one to two longitudinal modes within the Doppler broadened linewidth; the overtone UIUC SSL can accommodate four to five modes; the fundamental AVM can accommodate 11 to 12 modes. The fact that these lasers have a different number of possible lasing modes suggests that these lasers may saturate by different methods; perhaps homogeneously, perhaps inhomogeneously, or perhaps in some combination.

The possibility that overtone and fundamental lasers may saturate by different methods is supported by Mirels,²⁶ who showed that inhomogeneous broadening effects are expected to be negligible when

$$(\Delta\nu_h/\Delta\nu_d)R \geq O(10), \quad (2)$$

where $\Delta\nu_h$ and $\Delta\nu_d$ are the homogeneous and Doppler linewidths, respectively, and R is the ratio of the characteristic cross-relaxation rate to the characteristic collisional deactivation rate. For typical fundamental HF chemical lasers, Mirels showed that $(\Delta\nu_h/\Delta\nu_d)R = O[p(\text{Torr})]$, and relation (2) reduces to

$$p(\text{Torr}) \geq O(10), \quad (3)$$

where p is the static pressure in the lasing region in torr. Although the inhomogeneous effects are negligible when relation (3) is true, Mirels stated that

Table 1. Important Laser Linewidth Characteristics

Laser Wavelength (μm)	UIUC SSL		AVM
	Fundamental (≈ 2.82)	Overtone (≈ 1.36)	Fundamental Laser (≈ 2.82)
Pressure (Torr)	2.2	2.2	5.0
Temperature (K)	600	600	675
Lorentzian linewidth (MHz)	3.4	3.4	7.3
Doppler linewidth (MHz)	420	860	440
Mirror spacing (cm)	100	100	430
Mode spacing (MHz)	150	150	35
Number of modes	1-2	4-5	11-12

these effects do not become important in single longitudinal mode fundamental HF chemical lasers until

$$p(\text{Torr}) \leq O(1). \quad (4)$$

The fundamental UIUC SSL operates on one or two longitudinal modes (Table 1). However, because the pressure in the laser is approximately 2.2 Torr, relation (4) is not satisfied and Mirels's research shows that inhomogeneous broadening effects should not be very important for the fundamental UIUC SSL. This suggests that a mostly homogeneous gain saturation law is appropriate for the fundamental UIUC SSL. Because the AVM laser probably operates with 11–12 longitudinal modes (Table 1), this begins to approach the continuum limit and a mostly homogeneous gain saturation law is also appropriate for the fundamental AVM laser.

For the overtone laser, because $\Delta\nu_d$ is proportional to λ^{-1} , the Doppler linewidth of the overtone laser is approximately twice that of the fundamental laser because the overtone wavelengths are approximately a factor of 2 smaller than the fundamental wavelengths. Thus for typical overtone HF chemical lasers, it can be shown that $(\Delta\nu_h/\Delta\nu_d)R = (1/2)O[p(\text{Torr})]$, and relation (4) becomes

$$0.5p(\text{Torr}) \leq O(1). \quad (5)$$

Therefore, for the overtone UIUC SSL, a laser cavity pressure of 2.2 Torr satisfies relation (5) and inhomogeneous broadening effects should become important. From Table 1 we see that the overtone UIUC SSL probably operates on four to five longitudinal modes, which is not single-line inhomogeneous saturation but at the same time is not near the continuum limit; we should expect to see some inhomogeneous saturation effects.

A number of papers have used homogeneous Rigrod theory to make predictions about laser performance,^{17–24} but only Mirels and Batdorf²⁷ have presented any results that use an inhomogeneous gain saturation law. Mirels and Batdorf addressed the issue of the center-line intensity in an unstable resonator, but they made no comparison with data. To my knowledge, no papers have made a comparison between data and inhomogeneous saturation theory or have used a theory with a combination of homogeneous and inhomogeneous saturation.

Effects of Nonhomogeneous Gain Saturation

The general gain saturation law¹⁷ is

$$g = \frac{g_0}{[1 + (I/I_{\text{sat}})]^{1/m}}, \quad (6)$$

where g is the gain at some value of intensity, Ig_0 and I_{sat} are the standard Rigrod parameters, and m is what will be called the gain saturation law parameter. When $m = 1$, the gain saturation is called homogeneous as in Rigrod theory. When $m = 2$, the gain

saturation is called inhomogeneous. Common practice is to use homogeneous gain saturation in Rigrod theory because the equations can be solved analytically. The justification for using the homogeneous gain law for cw HF chemical lasers is that these devices typically run many longitudinal modes, and Milonni²⁵ showed that the saturation law for multiple modes in an inhomogeneously broadened medium has the same form as the homogeneous gain saturation law in the continuum limit. When $m = 2$, the equations must be solved numerically.

When Eq. (6) is expressed in Rigrod's notation (where $\beta = I/I_{\text{sat}}$) and the left and right running waves are taken into account,

$$g(z) = \frac{g_0}{(1 + \beta_+ + \beta_-)^{1/m}} \quad (7)$$

for the gain saturation law where β_+ and β_- are functions of z , the axial cavity position. The differential equations that must be solved are

$$g(z) = \frac{1}{\beta_+} \frac{d\beta_+}{dz} = -\frac{1}{\beta_-} \frac{d\beta_-}{dz}. \quad (8)$$

One relation that can be obtained from Eq. (8)¹⁸ is that

$$\beta_+\beta_- = C, \quad (9)$$

where C is a constant. Substituting Eqs. (7) and (9) into Eq. (8) gives

$$\frac{1}{\beta_+} \frac{d\beta_+}{dz} = \frac{g_0}{(1 + \beta_+ + C/\beta_+)^{1/m}}. \quad (10)$$

A nonsaturable uniform distributed loss term¹⁹ of $-\alpha_0$ could be added to the right-hand side of Eq. (10) for a more complete model. However, the results of treating the mirror losses as a distributed loss were not significantly different from the mirror loss results (Fig. 1), and the term is left out of the equation for simplicity. When $m = 1$ this differential equation can be integrated analytically¹⁸; however, numerical techniques must be used to solve Eq. (10) for other values of m .

As noted by Mirels and Batdorf,²⁷ the direct problem is to specify the two mirror reflectivities and then numerically integrate Eq. (10) until a solution is found. This requires an iterative procedure because the constant C is unknown. An indirect method suggested by Mirels and Batdorf²⁷ is to specify one mirror reflectivity and the value of β_+ at that mirror, e.g., let r_1 and β_1 be known values and let r_2 be unknown. By using the boundary conditions at the mirrors,¹⁸

$$\frac{\beta_2}{\beta_3} r_2 = \frac{\beta_4}{\beta_1} r_1 = 1, \quad (11)$$

one sees that, when r_1 and β_1 are known, so is β_4 . Because β_1 and β_4 are β_+ and β_- at $z = 0$, respectively,

the constant C can be calculated from Eqs. (9) and (11), i.e.,

$$C = \beta_1 \beta_4 = \beta_1 \frac{\beta_1}{r_1} = \frac{(\beta_1)^2}{r_1}. \quad (12)$$

The indirect problem then amounts to a straightforward numerical integration of Eq. (10) with g_0 , with C (determined from r_1 and β_1) and m given; no iteration is required. Once this numerical integration is completed, the value of $\beta_2 [= \beta_+(z = L)]$ will be known; from a similar manipulation of Eqs. (9) and (11), it can be shown that

$$r_2 = \frac{C}{(\beta_2)^2} = \left(\frac{\beta_1}{\beta_2}\right)^2 \frac{1}{r_1}. \quad (13)$$

Thus, through numerical integration of the indirect problem for a given r_1 , every value of β_1 corresponds to a value of β_2 and consequently r_2 . The intracavity problem is essentially solved at this point, and once the absorption-scattering losses of the mirrors a_x are given, the total outcoupled power can be computed from

$$P_{\text{out}} = I_{\text{sat}} A [\beta_4 (1 - r_1 - a_1) + \beta_2 (1 - r_2 - a_2)], \quad (14)$$

where A is the mode size of the beam and $1 - r_x - a_x = t_x$, the mirror transmissivity. For given r_1 , a_1 , a_2 , g_0 , I_{sat} , and A , by performing this integration over a range of β_1 , one can generate a curve of P_{out} versus r_2 [and equivalently P_{out} versus $g_t = -\ln(r_1 r_2)/2L$].

The numerical integration amounts to taking n small steps of size $\Delta z = L/n$, where L is the length of the lasing medium. A simple marching procedure was used to do the numerical integration, i.e.,

$$\beta_+(z + \Delta z) = \beta_+(z) + \frac{g_0 \beta_+(z) \Delta z}{[1 + \beta_+(z) + [(\beta_1)^2 / r_1 \beta_+(z)]^{1/m}]}, \quad (15)$$

where $\beta_+(z = 0) = \beta_1$. A nonsaturable uniform distributed loss term¹⁹ of $-\alpha_0 \beta_+(z) \Delta z$ could be added to the right-hand side of Eq. (15) for a more complete model. However, the results of treating the mirror losses as a distributed loss were not significantly different from the mirror loss results (Fig. 1), and the term is left out of the equation for simplicity. To check the numerical procedure's accuracy, curves of P_{out} versus r_2 for $n = 1000$ and $n = 10,000$ for $m = 1$ were generated and compared with the analytic solution; the results of both numeric solutions were accurate to within 0.2% of the analytic solution. Because computer CPU time was insignificant for these calculations on the mainframe Convex computer, double-precision variables and $n = 10,000$ was used for accuracy for all subsequent computations.

Once the accuracy of the numerical procedure is verified, it can be assumed that the accuracy is adequate to make computations for other values of m with confidence. The first cases run were for $m = 2$,

the inhomogeneous gain saturation law. For $m = 2$, the best agreement that could be obtained with data was found with $g_0 = 0.0325 \text{ cm}^{-1}$ and $I_{\text{sat}} = 25 \text{ W/cm}^2$ (Fig. 2). The value of I_{sat} is significantly different than the $m = 1$ (homogeneous) value of $I_{\text{sat}} = 175 \text{ W/cm}^2$; why is this so? When Eq. (6) is divided by g_0 and plotted for $m = 1$ and $m = 2$, the difference between the homogeneous and inhomogeneous gain saturation laws can be seen (Fig. 3). For the inhomogeneous case it is clear that significantly higher values of β are required to saturate the gain medium to the same level as that of the homogeneous case, e.g., to saturate to $g/g_0 = 0.50$, $\beta = 1.0$ when $m = 1$, but $\beta = 3.0$ when $m = 2$. For the same predicted output powers to be obtained, the intensities for the two cases must be the same. Therefore for $\beta (= I/I_{\text{sat}})$ to be larger for the inhomogeneous case, I_{sat} must be smaller. This explains why the inhomogeneous value of I_{sat} is significantly smaller than the homogeneous value of I_{sat} .

Figure 2 indicates that the $m = 1$ curve goes to a finite value [$P_{\text{available}} = g_0 I_{\text{sat}} AL$, Eq. (22) of Ref. 20] and that the modified Rigrod theory curves ($m \neq 1$) go to ∞ as $g_t \rightarrow 0$ (or equivalently as $r \rightarrow 1$ and as $t \rightarrow 0$). This of course is not a physical possibility and is merely a mathematical artifice of the idealistic Rigrod equations when no mirror or distributed losses are included. In the real world, every laser has mirrors with absorption-scattering losses and reflectivities less than unity and an intracavity (distributed) loss (although the intracavity loss in these HF lasers is negligible). When any of these losses are included (even extremely small losses less than 10^{-6}) into the $m = 1$ and $m \neq 1$ saturation theories, the total power goes to zero as $g_t \rightarrow 0$. All figures in this paper that have curves with losses show that the total power goes to zero as $g_t \rightarrow 0$.

From Fig. 2 it is clear that even the best fit with $m = 2$ is not a good match to data because the curvature of the inhomogeneous curve is too large. The fact that $m = 1$ has no curvature and $m = 2$ has too much curvature suggests the possibility that some intermediate value of m might yield a good

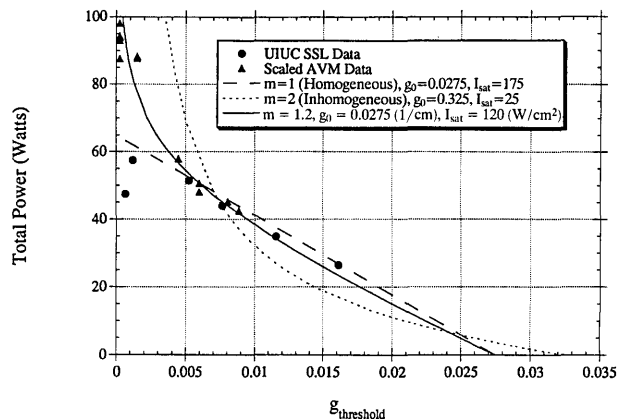


Fig. 2. Fundamental data versus Rigrod theory as a function of the gain saturation parameter m . The geometric gain lengths were used to compute g_t .

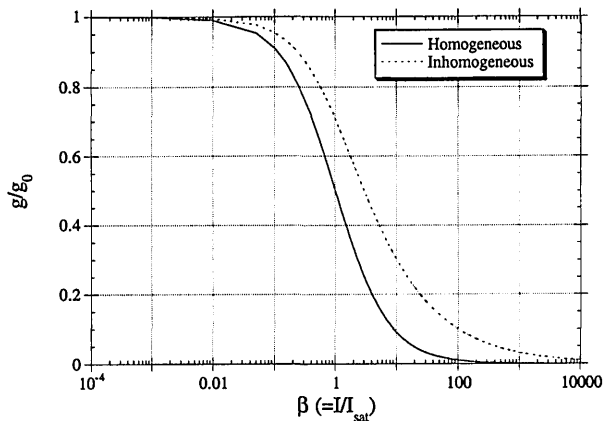


Fig. 3. g/g_0 versus $\beta (=I/I_{\text{sat}})$ for homogeneous and inhomogeneous gain saturation laws.

match with data. In fact, when $m = 1.2$ was used for the gain saturation law parameter, an excellent match to data was obtained for $g_0 = 0.0275 \text{ cm}^{-1}$ and $I_{\text{sat}} = 120 \text{ W/cm}^2$ (Fig. 2). When a 0.75% absorption loss was added for each mirror, the $m = 1.2$ Rigrod results curved downward in agreement with UIUC SSL data for small values of g_t (Fig. 4). This absorption of 0.75% is in better agreement with a recent absorption measurement of 0.63% ($\pm 0.03\%$ error in this measurement) for a fundamental enhanced total reflector than the previously determined loss of 0.25% with the homogeneous Rigrod equations. When the ORNECL results are plotted, there is very good agreement among data, the ORNECL model, and the modified Rigrod theory with $m = 1.2$ (Fig. 4). The result that an intermediate value of the gain saturation law parameter m gives good agreement with data is justified by the fact that the gain saturation in a real laser is not entirely homogeneous and not entirely single-mode inhomogeneous, but rather some combination of both. Figure 2 shows that the homogeneous and $m = 1.2$ curves follow roughly the same path until g_t drops below 0.005. The value of $m = 1.2$ that gives excellent agreement with data is much

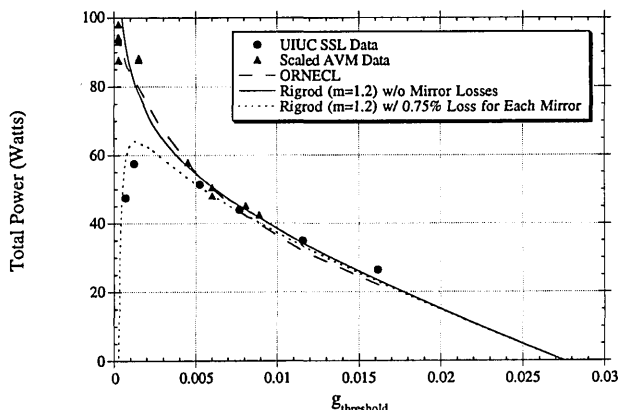


Fig. 4. Fundamental data versus the ORNECL model and Rigrod theory ($m = 1.2$, $g_0 = 0.0275 \text{ cm}^{-1}$, $I_{\text{sat}} = 120 \text{ W/cm}^2$) with and without mirror absorption losses. The geometric gain lengths were used to compute g_t .

closer to the homogeneous ($m = 1$) value than it is to the completely inhomogeneous ($m = 2$) value; this is reasonable because the *a priori* assumption was that the fundamental HF laser could be treated as if its gain will saturate approximately homogeneously, but clearly refutes it as an accurate assumption for all saturation levels.

Now that the $m = 1.2$ modified Rigrod theory matches the fundamental data with good accuracy for all saturation levels, different values of absorption-scattering loss for the fundamental mirrors can be input to determine what mirror characteristics are necessary to obtain outcoupled powers of 90–100 W. Figure 5 shows that a loss of 0.10% for each mirror is an acceptable absorption-scattering loss; a loss smaller than 0.10% is acceptable, but a loss of 0.15% would only give a maximum of 90 W. If one of the two mirrors is assumed to be 99.9% reflective, the outcoupler can have a reflectivity between 97.1 and 99.5% to obtain transmitted fundamental powers of 90–100 W; this is called a single-ended laser. Alternatively, this power level could be obtained with two identical transmissive optics having an absorption-scattering (AS) loss of 0.10% and reflectivities in the range 98.5–99.7%. This optical arrangement would outcouple equal amounts of fundamental power through each mirror and is called a symmetric laser. Ferguson and Latham²⁴ showed that the total outcoupled power from these two types of resonators (single ended and symmetric) is the same for low values of g_0L for $m = 1$ (the lowest value investigated in Ref. 24 was $g_0L = 2$). Because the value of g_0L for the fundamental UIUC SSL is 0.825 ($=0.0275 \times 30$), the results of Ferguson and Latham should apply for this lower value of g_0L . This was checked for $m = 1.2$ and $g_0L = 0.825$; there was less than 0.1% difference between the single-ended laser and the symmetric laser outcoupled power results. The essential mirror characteristic for either the single-ended laser or the symmetric laser is that the AS loss must be a maximum of 0.10%; otherwise the maximum P_{out} will not be obtained.

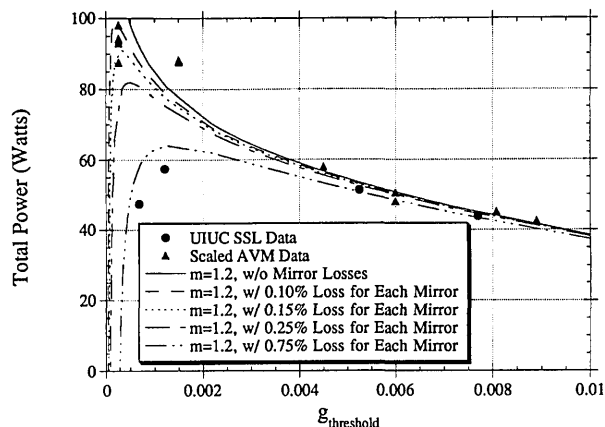


Fig. 5. Fundamental data versus Rigrod theory ($m = 1.2$, $g_0 = 0.0275 \text{ cm}^{-1}$, $I_{\text{sat}} = 120 \text{ W/cm}^2$) as a function of the mirror absorption losses.

The next step in this investigation was to determine if some value of m other than unity would also give a better match with existing overtone cw HF chemical laser data. First, as was done for the fundamental data, because the geometric gain length of 30 cm was to be used (approximately twice the effective mixed length of 14.9 cm), the homogeneous Rigrod curve²⁰ was rerun for a value of g_0 that was half the previous value, i.e., g_0 was decreased from 0.00085 to 0.000425 cm^{-1} . Two values of m were tried, $m = 1.2$ and $m = 2$. For each value of m , approximately the same ratios of $(g_0)_{m=1.2,2}/(g_0)_{m=1}$ and $(I_{\text{sat}})_{m=1.2,2}/(I_{\text{sat}})_{m=1}$ were taken for the overtone calculations as for the fundamental data, e.g., because $(g_0)_{m=2}/(g_0)_{m=1}$ for the fundamental calculations was 1.18 ($=0.035/0.0275$), $(g_0)_{m=2}/(g_0)_{m=1}$ for the overtone calculations was taken as 1.18 ($=0.0005/0.000425$). Figure 6 shows overtone data (corrected for absorption-scattering losses, Ref. 13) versus the Rigrod theory for three different values of m , $m = 1$ (homogeneous), $m = 1.2$, and $m = 2$ (inhomogeneous). The absorption-scattering loss of the mirrors was assumed to be zero in each of these calculations. Two sets of overtone data are plotted. One set used exclusively UIUC overtone mirrors coated by Rocky Mountain Instruments; the other set used one UIUC mirror in combination with one 4-in. (10.16-cm) TRW optic coated by Optical Coating Laboratory, Inc. From Fig. 6 it is clear that the $m = 2$ case gives the best agreement with the data. The $m = 1$ and $m = 1.2$ cases are not even close to the UI-TRW data points, whereas the $m = 2$ case has approximately the right shape to match the data even though it does not run through all of the UI-TRW data points. The $m = 1$ and $m = 1.2$ cases go to zero power at $g_t \approx 0.00042$, whereas the $m = 2$ curve runs through the UI-TRW data point at $g_t \approx 0.00045$ and does not go to zero power until $g_t \approx 0.00050$.

We ran ORNECL calculations for the overtone wavelengths to compare with data and the inhomogeneous modified Rigrod theory (Fig. 7). The ORNECL results do not rise as steeply as the data and inhomogeneous modified Rigrod theory. In contrast, the

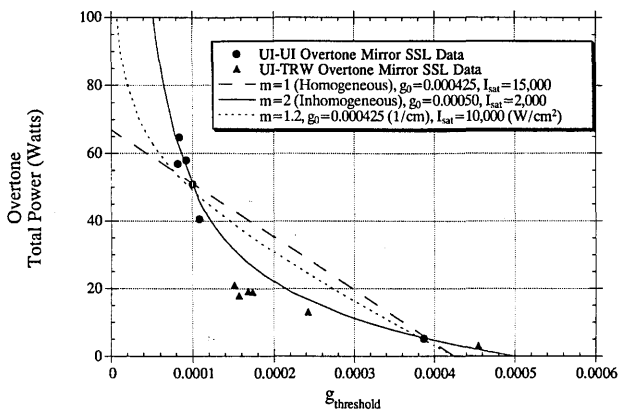


Fig. 6. Overtone power data versus Rigrod theory as a function of the gain saturation parameter m . The geometric gain length of 30 cm was used to compute g_t .

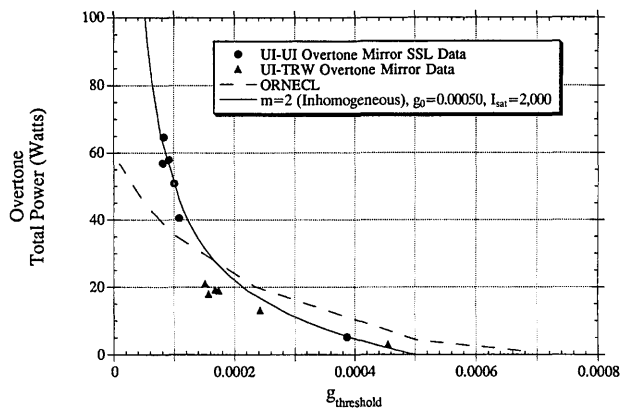


Fig. 7. Overtone data versus the ORNECL model and inhomogeneous Rigrod theory. The geometric gain length of 30 cm was used to compute g_t .

results of fundamental ORNECL calculations were in good agreement with the data and the mostly homogeneous ($m = 1.2$) Rigrod theory (Fig. 4). The reason why the ORNECL model predicts better agreement with fundamental data than overtone data is unknown.

As a verification that the inhomogeneous Rigrod theory predicted the actual outcoupled power for given overtone mirror sets,^{12,13} an iterative Rigrod code was developed in which the reflectivities and transmissivities of each of the two mirrors are specified; this is the direct problem mentioned earlier in this paper. Figure 8 shows that both the homogeneous and inhomogeneous modified Rigrod theories give good agreement with the outcoupled overtone power data. The $m = 2$ prediction for the lowest mirror loss point (20 W) is approximately 10% low, whereas the $m = 1$ prediction is approximately 20% low. The $m = 2$ prediction for the $AS_1 + AS_2 \approx 0.0052$ data point is approximately 10% high, whereas the $m = 1$ prediction is approximately 20% high. The $m = 1$ prediction for the $AS_1 + AS_2 = 0.0044$ data point is in better agreement with the data than the $m = 2$ prediction. This indicates that the inhomogeneous gain saturation law predicts existing overtone

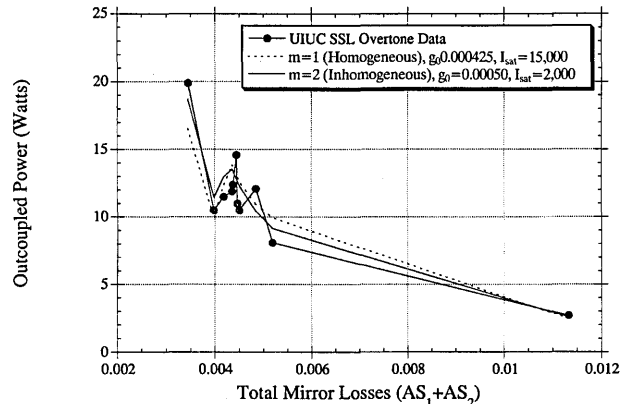


Fig. 8. UIUC SSL overtone data versus the inhomogeneous and homogeneous Rigrod theories as a function of the total mirror absorption-scattering losses.

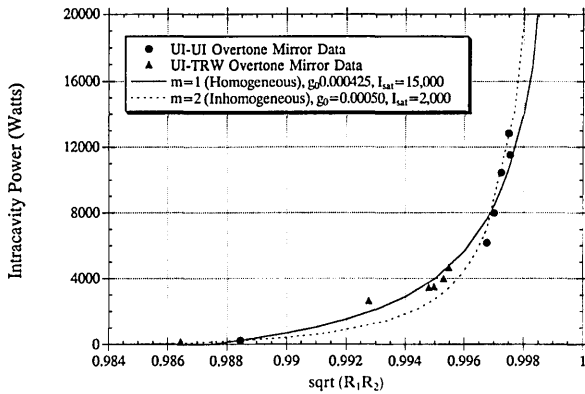


Fig. 9. Intracavity overtone power versus reflectivity for UIUC SSL data and the homogeneous and inhomogeneous Rigrod theories; sqrt, square root.

outcoupled power data as well as the homogeneous gain saturation law.

The next question to be raised is: If the inhomogeneous gain saturation law were used in the modified Rigrod theory, how would it affect the method of determining reflectivities of overtone mirrors outlined in detail in Ref. 20? In brief, this method consists of measuring the transmissivity and outcoupled power from each of a pair of overtone mirrors (these are simple and quick laboratory measurements); then the one-way intracavity power can be calculated [$P_{IC} = (P_1 + P_2)/(T_1 + T_2)$]. Using a Rigrod plot of average one-way intracavity power [$P_{IC} = I_{sat}A(\beta_1 + \beta_4)/2$] versus $(r_1r_2)^{1/2}$, one can read off the value of $(r_1r_2)^{1/2}$ for the set of mirrors in question. If one mirror reflectivity is already known, then the other can be calculated. To answer the above question, a plot of P_{IC} versus $(r_1r_2)^{1/2}$ was generated for both the homogeneous and inhomogeneous gain saturation laws for the overtone (Fig. 9). Figure 9 shows that there is no large difference in the magnitude or shape of the predicted P_{IC} versus $(r_1r_2)^{1/2}$ curves. If the $m = 2$ curve was used it would shift the predicted TRW mirror reflectivities from approximately 0.993 to approximately 0.994, which is a 0.001 change. Because the accuracy of the previously reported measurements made by UIUC for high-reflectivity overtone mirrors is ± 0.0007 ,²⁰ the change in using the inhomogeneous curve is not far outside the error bounds of the reflectivity estimates made by using the homogeneous curve. Because this is not a large discrepancy, until more overtone data are obtained in the reflectivity range of 0.992–0.996, the UIUC will continue to use the homogeneous Rigrod results for determining overtone mirror reflectivities.

Concluding Remarks

A modified Rigrod theory that uses a nonhomogeneous gain saturation law was used to model outcoupled, total, and intracavity power from a high-gain and a low-gain cw HF chemical laser with good accuracy. From scaled HF AVM data, the homogeneous gain saturation law was found to have signifi-

cant limitations when used to model total power data over a wide range of threshold gain values. The use of a nonhomogeneous gain saturation law indicated that a gain saturation law parameter of $m = 1.2$ models AVM and UIUC SSL fundamental HF data more accurately than the completely homogeneous value of $m = 1$ with mirror losses or a distributed loss. A completely inhomogeneous saturation law ($m = 2$) models existing UIUC SSL overtone data more accurately than the homogeneous law ($m = 1$) with mirror losses or a distributed loss. The prediction of overtone mirror reflectivities as outlined in Ref. 20 is the same using either the homogeneous or inhomogeneous saturation laws within the experimental uncertainty of the data.

Scaled AVM data taken at a lower threshold gain than UIUC SSL data indicates that the maximum UIUC SSL fundamental power may be approximately 94 W rather than the previously reported 76 W. Thus the scaled AVM data suggest that the maximum measured UIUC SSL overtone efficiency may be 68%, which is closer to the previously estimated lower bound of 70% than to the upper bound of 90%.

The use of the $m = 1.2$ gain saturation law for the fundamental data permits the modified Rigrod theory to be used to predict the mirror characteristics required to obtain significantly higher outcoupled fundamental powers. As a verification that the maximum SSL fundamental power may be as large as 94 W, it will be necessary to obtain two fundamental mirrors with a maximum AS loss of 0.10% and reflectivities in the range 98.5–99.7%.

Appendix A. Notation Used in This Paper

A	Beam area or mode size (cm ²)
a_x, AS_x	Absorption–scattering loss of mirror x
f_v	Fraction of HF molecules pumped into vibrational level v
g	Gain (cm ⁻¹)
g_0	Rigrod gain coefficient parameter (cm ⁻¹)
g_t	Threshold (or saturated) gain during lasing conditions (cm ⁻¹)
I_{sat}	Rigrod saturation intensity parameter (W/cm ²)
L, L_g	Total gain length of nozzle bank, $L_g \geq L_e$ (cm)
L_e	Effective length of the mixed flow (cm)
m	Gain saturation law parameter
P_{AS}	Absorbed–scattered power [$= (P_{AS})_1 + (P_{AS})_2$]
P_{IC}	Intracavity power (one way)
P_{out}	Outcoupled (transmitted) power [$= (P_{out})_1 + (P_{out})_2$]
P_{total}	Total transmitted–absorbed–scattered power ($= P_{out} + P_{AS}$)
r_x, R_x	Reflectivity of mirror x
t_x, T_x	Transmissivity of mirror x
α, α_0	Nonsaturable uniform distributed loss parameter (cm ⁻¹)
β_i	Normalized intensity used in the Rigrod theory ($= I_i / I_{sat}$)

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